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Problem Chosen

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Summary Sheet

Summary

With the increase in concerns over air quality and energy consumption, electric buses (e-buses) have emerged as a compelling solution. Given the increase in conversion from traditional diesel buses to electric buses around the world, it is important to analyze its environmental consequences and economic implications, and develop plans for the transition of electric buses.

Firstly, we collected data for greenhouse gas emissions (CO_2, CH_4, N_2O) and noise pollution to analyze the environmental consequences of converting to an all-electric bus fleet. Using 90% of the data as the training set and 10% of the data as the test set, we used the logistic regression model for noise pollution; linear, improved logistic, and exponential models were used for greenhouse gas emission. Each model has a R-square value of 0.944, 0.884, 0.889, and 0.932, respectively. By comparing the latter 3 models for CO2 emission, we chose to use the logistic model since it is more accurate based on its R-square value, Root Mean Square Error, Mean Absolute Percentage Error, and its trend is more consistent with the reality. After applying the models for noise pollution and CO2 emission to the city of Seattle, we found that if 100% of its buses are electric buses, then the greenhouse gas emission will decrease from 1.416×10^{11} grams (currently) to 6.252×10^{10} grams; for noise pollution, it will increase from 1.183 times lower to 3.308 times lower than that of a fully diesel bus fleet.

To assess the financial implications of converting to e-buses, we constructed a recursive equation model using the number of electric buses in year t and the number of transitions this year to determine the total number of electric buses in year $t+1$. To calculate the number of transitions, we take usable profit, external funding, operational costs of all buses, and conversion costs of electric buses into consideration to determine the total number of electric buses. By applying this model to Seattle, which has 1464 buses in total (including 185 electric buses), we found that it will take around 6 years, if 50% of the transition cost is covered by funding, to convert all of it buses to electric buses. However, funding covering 20% of the cost will already be sufficient for Seattle to achieve a fully electric bus fleet within 10 years.

Then, we developed a 10 year plan for converting all buses to electric buses, and applied the models to Seattle, San Francisco, and New York. We used the Entropy Weight Method and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to determine which bus route should be transformed first, which takes the number of ridership, route length, and elevation of the road into consideration. In the result, we displayed the entropy weight of the indicators in each city, as well as the rankings of bus routes in each city. Next, to determine the locations of the charging stations for electric buses, we employed the K Mean Clustering Model to determine the location that is most convenient for buses to charge. The coordinates of the charging stations are presented in the result. Lastly, we constructed a sensitivity analysis based on our models.

Keywords: Electric bus, Regression model, Entropy Weight Method, TOPSIS

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1 Introduction

1.1 Background

Electric buses (e-buses) have been introduced since the 20th century, but due to limited technology and high costs at that time, gasoline-run vehicle was a more appealing option. E-buses can store their electricity on board, primarily using rechargeable batteries, or they can draw power from external sources such as overhead wires and ground-level power supply systems. In recent years, improvements in factors like battery technology, charging infrastructure, and sustainability have led to a surge in the adoption of electric buses, especially in China. In 2017, China accounted for 99 percent of the 385,000 electric buses worldwide, which is 17 percent of China's total bus fleet. Other regions around the world are also taking gradual steps to incorporate electric buses. In Western Europe, for instance, the number of electric bus registrations had tripled in 2019, and continuously increased in subsequent years. In 2021, the number of electric registrations increased 48% compared to 2020.[1]

The rise in carbon dioxide (CO_2) concentration in the atmosphere is the main driving force of global warming. According to the International Energy Agency, the transport sector contributes to nearly a quarter of global energy-related CO_2 emissions [2], making it a crucial role in mitigating the issue of climate change. One notable solution is the substitution of diesel buses with electric buses, which can result in a substantial reduction in CO_2 emissions and energy consumption. Unlike traditional diesel buses, e-buses rely solely on electricity for power, emitting no pollutants from their tailpipes. This reduction in emissions also has a positive impact on air quality, which could lead to a decrease in health-related issues. E-buses' ability to recover energy through regenerative break also allows them to be more energy efficient compared to diesel buses. Further, they help reduce noise pollution and have lower operating costs when compared to their diesel counterparts. However, there are also challenges including high purchase price, limited range, and the need for charging infrastructure.

In this contest, we will create models to understand the environmental consequences and economic costs associated with transitioning to an electric bus fleet. We will also provide a 10 years plan to convert buses into fully electric bus fleet.

1.2 Problem Restatement

In this study, we aim to accomplish the following tasks.

Question 1: Construct a model that aids cities in understanding the environmental impacts of shifting to a fully electric bus fleet. Apply the model to a metropolitan area with a population exceeding 500,000 people that presently lacks a fully electric bus fleet.

Question 2: Construct a model that focuses on the economic implication of transitioning to e-buses. It should take potential external funding that covers up to half of the transition costs into consideration. Apply the financial model to the same area from the previous analysis.

Question 3: Use the previously developed models (or develop new ones) to create a 10-year roadmap that urban transport authorities can use to plan their e-bus fleet updates. The goal is to convert buses to a full electric bus fleet by 2033. Apply your models (or create new ones) to 3 area (including the same area used previously).

2 Assumptions

- **Assumption 1**: US buses, except for e-buses, are all diesel buses.
  Justification: According to Statista, in 2020, 96.2% of the transit bus fleet in the United States were diesel buses [3]. Since the other fuel types only accounted for a very few percentage of the total fuel types, we will not take them into consideration.

- **Assumption 2**: Electric buses transform yearly instead of continuously.
  Justification: Companies typically review their performance through annual financial reports, and since the scale of converting to e-buses depends on information from these yearly reports, the transformation is assumed to occur on a yearly basis.

- **Assumption 3**: There won't be a sudden drop in passengers caused by uncontrollable factors, such as the pandemic.
  Justification: Since these types of events are unpredictable and happen on a very low chance, we will not take them into account in this paper.

- **Assumption 4**: The number of buses before and after conversion remains constant. **Justification**: If there is an increase in the total number of buses, issue of storage and land occupation may appeared. Even if there is an increase in the number of buses, the amount will be unpredictable, therefore, we assume that the remains the same before and after the transition to e-buses.

3 Data and Variables

Symbol	Definition
B	Total number of buses
E_B	Total number of electric buses
D_B	Total number of diesel buses
E_{mi}	Number of miles electric buses traveled
D_{mi}	Number of miles diesel buses traveled
$E\%$	Percentage of electric buses
E_n	Noise of electric buses
D_n	Noise of diesel buses
dB_{df}	Decibel difference
CO_2/mi	CO_2 Emitted per mile
E_t	Total number of electric buses in t year
E_0	Total number of electric buses in base year
$k\%$	Safe proportion of total profit that companies invest in the conversion of e-buses
$F\%$	Percentage of cost covered by external funds
N_T	Times of noise quieter
CS	Number of Charging Ports

We first collected data from 103 cities in the United States from the 2021 Revenue Vehicle Inventory published by the Federal Transit Administration, and calculated variables including the percentage of electric bus fleet, the percentage of electric bus miles, and the diesel bus miles in each city[4]. 93 US cities will be used as the training set, while 10 cities will be used as the test set.

Regarding greenhouse gas emissions for diesel buses, we found the emission per mile of CO_2, CH_4, and N_2O, which is 2680 g/mi, 0.0051 g/mi, and 0.0048 g/mi respectively [5][6]. To find the total amount of greenhouse gas emitted by diesel buses, we multiply the emission per mile of each gas by the total miles the bus traveled. We then add the total emission of each gas together and get the total greenhouse emission.

Since electric buses emit zero tailpipe pollution, we take the emissions produced by the electric supply factories into consideration. To collectively calculate the amount of greenhouse gases emitted by electric buses, we convert the emissions of $CH4$ and N_2O into the equivalent amount of CO_2 emission by using the Global Warming Potential (GPW), which is the measure of the amount of greenhouse gas emission relative to CO_2. The GWP ratio for CO_2: $CH4$: N_2O is 1: 25: 298 [7].

To measure the noise pollution of buses, we found the decibels (dB) of diesel buses and electric buses, which are around 80 dB and 63 dB respectively [8].

4 Problem 1: Ecological Consequences Model

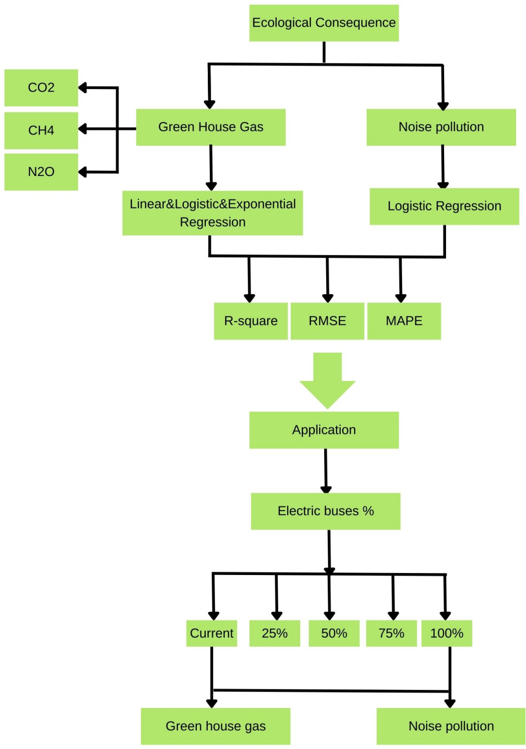

Figure 1: Overall Idea of Question 1

In order to understand the ecological consequences of transitioning to an all-electric bus fleet, we constructed three main models: linear, logistic, and exponential model. We take the percentage of transition as the independent variable and the ecological consequences (including CO_2, CH_4, N_2O, and noise pollution) as the dependent variables. Then we compare these models and select the more accurate and reasonable one, which will be applied to calculation of greenhouse gas emissions and noise pollution for our chosen city. The overall idea of question 1 is shown in Figure 1.

4.1 Noise Pollution Determination

We first constructed an equation to find the difference in noise, or decibel difference (dB_{df}) between a current city with both diesel and electric buses and a hypothetical city with all decibel buses. This allows us to determine how much quieter a city is compared to a city will all decibel buses. The equation is shown below:

$$dB_{df} = D_n - (D_n(1 - E\%) + E_n E\%) \tag{1}$$

Based on our data collected, the decibel for a diesel bus is 80 dB while the decibel for an electric bus is 63 dB. Hence, in the equation shown above, D_n and E_n each corresponds to 80 and 63, and $E\%$ denotes the percentage of electric bus in the city.

Due to a nonlinear relationship between decibels and the perceived loudness, decibels cannot be directly transformed into the noise heard. Thus, we used the standard of the noise difference where difference of 10 dB is 2 times the perceived loudness, and the difference of 20 dB is 4 times the perceived loudness.

$$N_T = 1.0198e^{0.0693DBF} \tag{2}$$

Then, we create a best-fit line using the total difference in decibels to calculate how many times quieter the noise is when all buses in a city are diesel buses. In equation (2), N_T represents how many times the noise is quieter, and dB_{df} represents the decibels difference, which is calculated through equation (1).

4.1.1 Logistic Regression Model

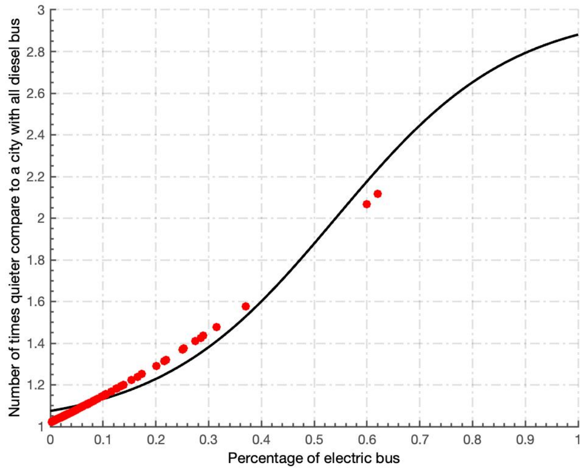

Figure 2: Logistic Regression for Noise Pollution

From the collected data, we plot a scatter graph, as seen in Figure 2. We observe the relationship between how many times quieter the noise is and the percentage of electric buses. Considering there is an upper limit and a lower limit of the number of times noise quieter, we choose to construct a logistic model instead of a linear regression:

$$N_T = \frac{2}{1 + a_0 e^{-a_1 E\%}} + 1, R^2 = 0.944 \tag{3}$$

where a_0 and a_1 are two coefficients, and N_T denotes to how much times quieter the noise is.

Based on the dB standard, 20 dB difference means 4 times the noise. Since the decibel difference between electric bus and diesel bus is 17 dB, we can conclude that a 17 dB difference will always be 4 times lower the noise. As shown in the graph, the upper bound did not reach 4, which is consistent with our estimation.

4.2 CO_2 Emission Determination

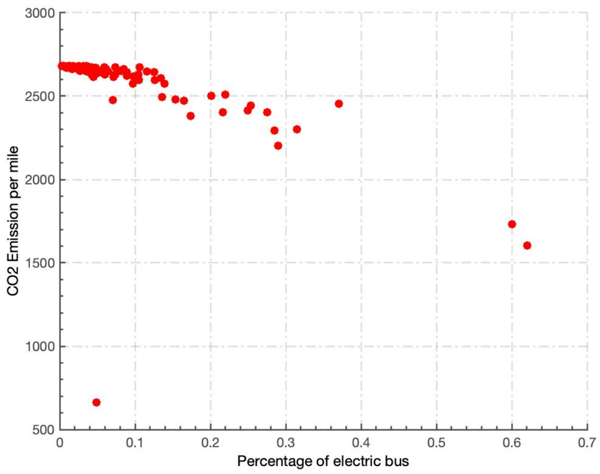

Figure 3: Scatterplot for CO2 Emission

The second factor of ecological consequences we consider is the amount of greenhouse gas emitted by diesel buses. To calculate the total emission of different types of greenhouse gases, we converted different pollutants, namely NH_4 and N_2O, into CO_2 by using global warming potential ratio, as mentioned in the data section, so that we can collectively call these three greenhouse gases emission as CO_2 emission. For CO_2 emission, we decide to construct 3 models: Linear regression, Improved Logistic regression, and exponential model. Then, we will analyze their results and apply the most accurate one to the selected city. We calculated the percentage of electric buses and the amount of CO2 emission in each city, and plotted the following graph. Since the majority of US cities have a small proportion of electric bus fleet, most of the data points are clustered at the left side of the graph.

4.2.1 Linear Regression Model (Least Square Regression)

To determine the relationship between the percentage of electric fleet and the pollution emitted, we first created a least square regression model:

$$CO_2/mi = 2718.957 - 1423.922E\%, R^2 = 0.884. \quad (4)$$

The total CO_2 emission per mile is the dependent variable while the percentage of electric buses is the independent variable. We draw a least square regression line based on the formula shown above and can see that a higher percentage of electric fleet correlates with a lower amount of carbon dioxide emission per mile.

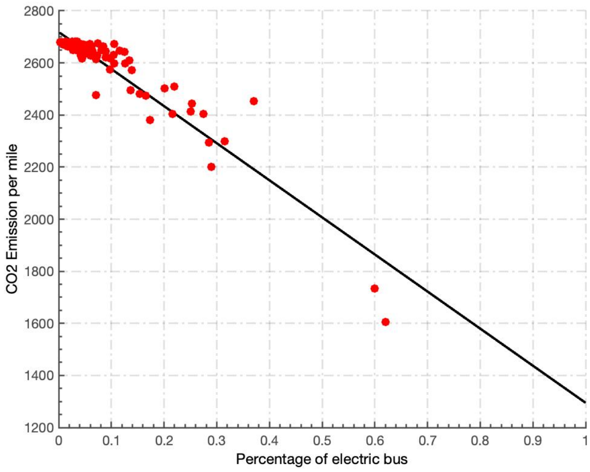

Figure 4: Linear Regression for CO_2

The reason that CO_2 emission is not zero even when all of the buses are electrical is because we also take the amount of CO_2 emitted by the factory that produced electric buses into consideration.

4.2.2 Improved Logistic Regression Model

We then create an improved logistic regression model that better fits with our data. Since the dependent variable represents the percentage of electric buses and the independent variable is the amount of CO_2 per mile, using a regular logistic regression will show a directly proportional relationship between x and y, which is not what we wanted. Therefore, we improve the logistic regression model by reflecting it through the y-axis and shifting it upward. As a result, an increase in the percentage of electric buses will cause the CO_2 emission per mile to decrease, which fits with the logic.
The improved logistic regression model is shown below:

$$CO_2/mi = \frac{1581}{1 + 0.024e^{7.282E\%}} + 1100, R^2 = 0.889 \quad (5)$$

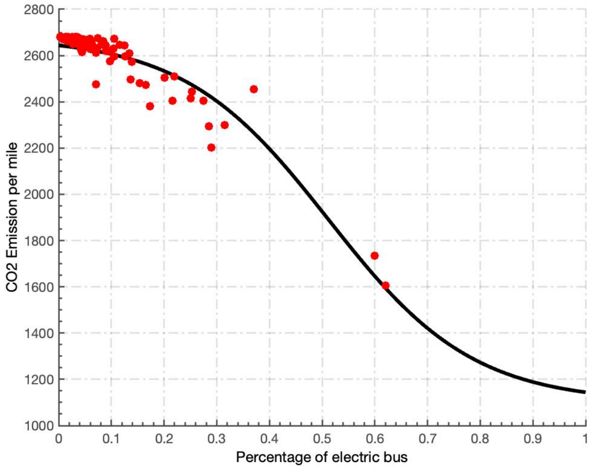

Figure 5: Improved Logistic Regression Model for $CO2$

4.2.3 Exponential Regression Model

We also use the exponential regression model to find the correlation between percentage of electric buses and the amount of CO_2 emission per mile, as shown in the equation below:

$$CO_2/mi = -377.032e^{2.137E\%} + 3066.327 \tag{6}$$

The reason that an exponential model might fits with our data is because by looking at the scatter plot, it's possible that as percentage of electric bus increase, the amount of CO_2 emitted may decrease exponentially.

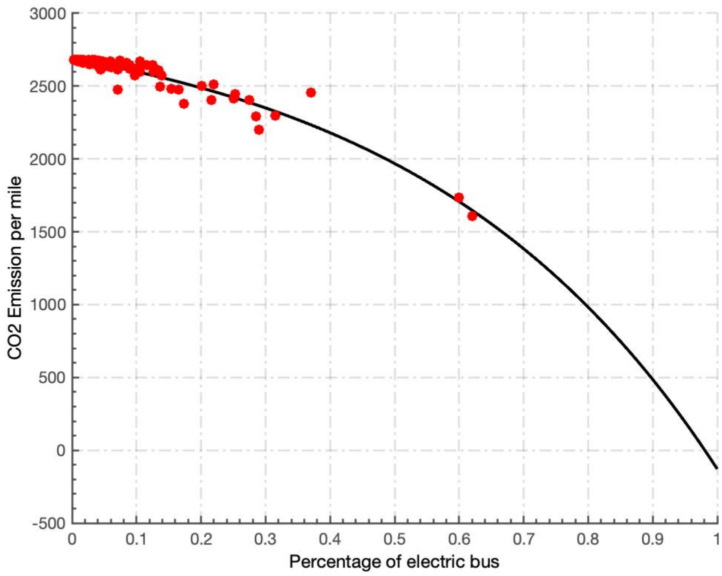

Figure 6: Exponential Regression Model for $CO2$

4.3 $CO2$ Models Comparison

We first analyzed the results of the 3 regression models and see if each is valid based logic. The table below shows the results of the three models. y_L, y_E, and y_{linear} represent the CO_2 emission per miles for improved logistic model, exponential regression model and linear regression model, respectively. X represents the percentage of electric bus transition. There is a notable difference between the carbon emission value of exponential model and the other two models. As shown on the table, for exponential model, at 100 percent electric buses, CO_2 is -128.63g/mile, which is impossible to achieve in real life. As a result, the exponential model does not seems to be a suitable model.

x	y_L	y_E	y_{linear}
0%	2719.88	2689.30	2718.96
100%	1053.35	-128.63	1295.04

Table 1: Table to test captions and labels.

Then, we compare the root mean square error and mean absolute percentage error of the three models. Firstly, we compare the R Square of two models to see which model has a better performance. About 88.9 percent of the variability in CO_2 emission per mile is accounted for by the least square regression model, while about 88.4 percent of the variability in CO_2 emission per mile is accounted for by the logistic regression model. Exponential regression model has a r square value of 0.932, which is higher than that of the logistic model. However, we found that exponential model isn't a suitable model for our case since the CO2 emission per mile becomes negative when there are 100% electric buses, which is not possible. Therefore, we selected the logistic regression model because it has a higher R-Square value compared to the linear regression model. Also, it is logically more reasonable than the exponential regression model.

Model	Logistic	Exponential	Linear
R-Square	0.882	0.932	0.889
RMSE	33.478	35.781	213.061
MAPE	1.167	0.481	6.367

Table 2: Table to test captions and labels.

We also compare the root mean square error (RMSE) for the three models to better evaluate which model is better.[9]

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{7}$$

We use 10 experimental groups to calculate the mean square error. The result shows that the logistic regression model has a mean square error of 33.478 and the Linear regression model has a mean square error of 213.061, as shown in the table above. It's clear that linear regression models have a higher root mean square error, and have a much wider spread of the data. This shows that compared to the Logistic regression model, the Linear regression model is much more unstable and far away from the actual experimenting group.

We then use mean absolute percentage error to compare 3 model, as shown below:

$$MAPE = \frac{100}{n}\sum_{i}^{n}|\frac{y-\widehat{y}}{y}| \tag{8}$$

As shown in the table above, logistic regression model have a MAPE of 1.167 while linear regression model have a mean absolute percentage error of 6.367. Since lower MAPE is inaccurate, we choose logistic regression model as our final model. Eventhough exponential regression model have a MAPE of 0.481, which is very accurate. However, since we found out that exponential regression model have invalid result, we eliminate it.

4.4 Application to Seattle

We then apply the logistic regression model of noise and improved logistic regression model of CO_2 into the city of Seattle. We separate the conversion of electric bus into 4 stages: 25%, 50%, 75% and 100% and observe how each stage will influence the noise pollution and CO2 emission.

Result	Current (12.6% E_B)	25% E_B	50% E_B	75% E_B	100% E_B
CO2 Emission	1.416×10^{11}	1.354×10^{11}	1.052×10^{11}	7.311×10^{10}	6.252×10^{10}
Noise level lower	1.183	1.369	1.834	2.465	3.308

Table 3: CO2 Emission and noise level (times lower than a full diesel bus fleet)

4.4.1 Noise Pollution

The current noise level of buses in Seattle will be around 1.18 times lower than Seattle whose buses are all diesel buses. When there are 25 %, 50 %, 75 %, and 100 % electric buses in Seattle, the noise level will be around 1.37 times, 1.84 times, 2.46 times, and 3.3 times lower respectively than when all buses in the city are diesel buses. This shows that an increase in the percentage of electric buses can effectively reduce noise pollution in Seattle. Compared to the current noise level in Seattle, which is 1.183, the all-electric bus is 3 times quieter than the current noise level.

4.4.2 CO_2 Emission

Based on our improved logistic model, the current total greenhouse gas emission by all buses in Seattle will be 1.416×10^{11} g/yr. If 25 percent, 50 percent, 75 percent, or 100 percent of Seattle's buses are electric, then will have an emission of 1.35367×10^{11} g/yr, 1.05232×10^{11} g/yr, 7.3108×10^{10} g/yr, and 6.25232×10^{10} g/yr respectively.

4.5 Sensitivity Analysis

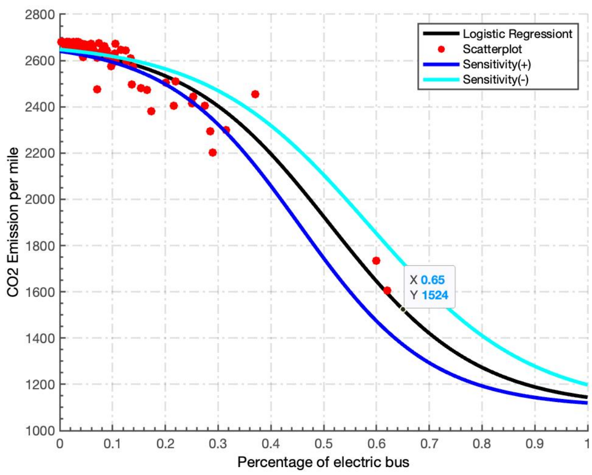

Figure 7: CO_2 Improved Logistic regression Sensitivity Analysis

Lastly, we analyze the sensitivity of our improved logistic model by changing a_0 and a_1 by ±10%, so we get $a_0 = 0.024 \pm 0.0024$, and $a_1 = 7.282 \pm 0.7282$.

When we set $E_p = 0.1$, the original result is 2763.71. After changing a_0 or a_1 by 10%, the result is 2772.46 or 2770.31, which only changed by 0.24%. Therefore, we consider our model to be stable and relatively insensitive to abnormal data.

5 Problem 2: Financial Model

For problem 2, we construct a mathematical model to analyze the financial implications associated with transitioning to e-buses, specifically the recursive model. We assume that the scale of transformation to e-buses is based on the number of electric buses in the previous year, which is influenced by the company's decision to allocate a certain proportion of its profit as an investment for the conversion. Given the dynamic nature and variability of transitioning to e-buses, a recursive model is suitable due to its ability to continuously update and adapt to changing condition each year.

5.1 Recursive Function Model

To determine the total number of electric buses in year t, denoted as E_t below, we add the total number of electric buses in the base year (E_0) with the calculated number of added electric buses in this year (ΔE_t):

$$E_t = E_0 + \Delta E_t \tag{9}$$

When a bus agency needs to convert their diesel bus to electric bus, they need to invest money into the project every year. Hence, we calculate the number of new electric buses that may be converted in this year by the following equation:

$$\Delta E_t = \frac{P}{C_{peb}} \tag{10}$$

The amount of electric bus transit in one year depends on the profit the agency generated from the buses in the previous year, which can be calculated through the general equation $Profit = Revenue - Cost$. We assume that the revenue for electric buses and diesel buses will be the same, because citizens are not likely to pay for additional ticket prices for electric buses. However, not all of the profit earned would be used in investment since some revenue might be used to counter risk in the future. So the usable capital is only a proportion of the total profit generated, and the proportion that were used to invest is denoted as $k\%$:

$$\Delta E_t = \frac{k\% \cdot R_{tot} - C_{tot}}{C_{peb}} = E_t - E_0 \tag{11}$$

As shown in equation above, R_{tot} is total revenue generated from the year $t_0$to year $(t-1)$. C_{tot} is total running cost of all the buses, and C_{peb} denotes as the cost per additional electric bus and infrastructure such as charging port.

Since we assumed that buses are only composed of electric and diesel bus, the total cost(C_{tot}) is made up of the running cost of diesel buses (C_D), the running cost of electric buses (C_E), and the total cost of bus drivers' salaries (C_S):

$$C_{tot} = C_D + C_E + C_S \tag{12}$$

Each cost is calculated through the following equations:

$$C_D = D \cdot D_d \cdot C_{dpm} \tag{13}$$

$$C_E = E_0 \cdot D_e \cdot C_{epm} + \Delta E_t \cdot D_d \cdot C_{epm} \tag{14}$$

$$C_S = B \cdot C_{spb} \tag{15}$$

B is the total number of electric and diesel bus in current base year, and $B = D + E_0$, where D and E_0 denotes the number of diesel buses, and number of electric buses in base year, respectively. C_{spb} represents the average annual salary per bus driver. D_d and D_e denotes the average distance, in miles, traveled by one diesel bus or one electric bus in one year, respectively. C_{dpm} represents the total cost of one diesel bus for each mile driven. This includes the cost of fuel which is diesel in this case, measured in dollar per gallon, and the cost for maintenance, such as cleaning engine and changing the brakes, measured in dollar per mile.

Similarly, C_{epm} represents the total cost of one electric bus for each mile driven, but in this case, the fuel used is in the form of electricity. Moreover, the maintenance cost of electric bus driven per mile would be much lower than of diesel bus. This is because electric bus does not require changes of oil or air filter, and its mechanism of regenerative braking that only apply the traditional friction brakes when it is slow enough, the fuel efficiency is maximized and cost of changing brakes are reduced by half. Although there is also a cost for changing battery for electric bus, most of electric bus agencies now provide a 10-year warrant, so we did not take the cost of changing batteries into account.

Since E_0 is a constant number, and the result of the part $E_0 \cdot D_e \cdot C_{epm}$ is the cost of all electric buses in the base year, we add the cost of newly converted electric bus in year t through $\Delta E_t \cdot D_d \cdot C_{epm}$. Here, we use D_d, the average distance driven by a diesel bus, instead of D_e because the new electric buses will follow the original path of the diesel buses, meaning there is no change in average distance traveled per bus. The recursion also occur in this part where ΔE_t is used inside itself.

For C_{peb}, we take the upfront purchasing cost of one electric bus (C_b) and the cost of one charging station (C_c) into consideration. Since an addition in one converted electric bus does not mean addition in one new charging station, we set ρ to denote the ratio between number of electric buses and number of charging ports. Then, we consider the external funds being provided, which is denoted as $F\%$. This funding varies from 0% to 50%. We multiply the cost per electric bus with the actual percentage company need to self-invest, which is resulted from $1 - F\%$:

$$C_{peb} = (C_b + \rho \cdot C_c) \cdot (1 - F\%) \tag{16}$$

Lastly, putting all of the above equations altogether, we construct a final model to find the total number of electric bus t years from the base year:

$$E_t = E_0 + \frac{k\%(R_{tot}(t-1) - \sum_{i=0}^{t-1}(D_B D_d C_{dpm} + E_0 D_e C_{epm} + (E_i - E_0) D_d C_{epm} + B \times C_{spb})}{(C_b + \rho \cdot C_c)(1 - F\%)} \tag{17}$$

where $E_i - E_0$ is derived from ΔE_t.

5.2 Parameter Determination

According to the source, the safe proportion of the profit that a company should use to invest into the conversion project is from 20% to 25%.[10] However, since we are applying the model to Seattle where there is only one large bus agency, King Country Metro, which is a government agency, we take into account of its unnecessity of paying taxes. Thus, we set $k\% = 0.5$, adding 30% to usual safe proportion investment. From the calculation of average number of passenger per day times ticket fee times 365, we get $R_{tot} = \$326,036,250$. [11]

From the previous 95 data, we found the total number of bus in Seattle in 2021 is 1467 and number of electric bus is 185. We assumed that the total number of bus will not change as diesel bus is converting to electric us, so we set $N_{tot} = 1467$, $E_0 = 185$, and $D_t = 1467 - E_{t-1}$, so $D_t = 1282$.

Based on the data, we set $D_d = 40386.5$ miles/yr, $D_e = 15542$ miles/yr, $C_{spb} = \$44501$, $C_{dpm} = \$1.9$ per mile, with the fuel costing 0.76 dollar per mile and 1.14 dollar per mile for maintenance cost, and $Cepm = 1$, with the fuel of 0.36 dollar per mile and maintenance cost of 0.64 dollar per mile.[12][13][14] The data also gives $C_b = \$797822$ and $C_c = \$60000$.[13] Lastly, we set $\rho = 0.25$ because a charging port allows four electric buses to charge at the same time.[15]

5.3 Result Analysis

	50%	40%	30%	20%	10%
1	381	348	325	307	294
2	586	517	469	433	405
3	800	692	618	562	519
4	1023	874	771	695	636
5	1256	1063	929	831	756
6	1500	1259	1092	971	879
7		1462	1261	1115	1005
8		1673	1435	1263	1134
9			1615	1415	1266
10				1572	1401
11					1540

Table 4: Prediction of Seattle conversion to electric buses with different percentage fund

We wrote this recursion model in Java, as shown in the appendix, and entered data from Seattle in 2021. The predicted number of years needed to convert 1282 buses in Seattle into electric buses (1464 buses in total) under different percentage of cost being covered by funding is shown in the following table:

From Table 4, we can observe that with 50% of cost covered by external funding, Seattle could achieve an all-electric bus city within six years. In contrast, if the external funding percentage is only 10%, it would take a total of eleven years to convert all buses to electric power.

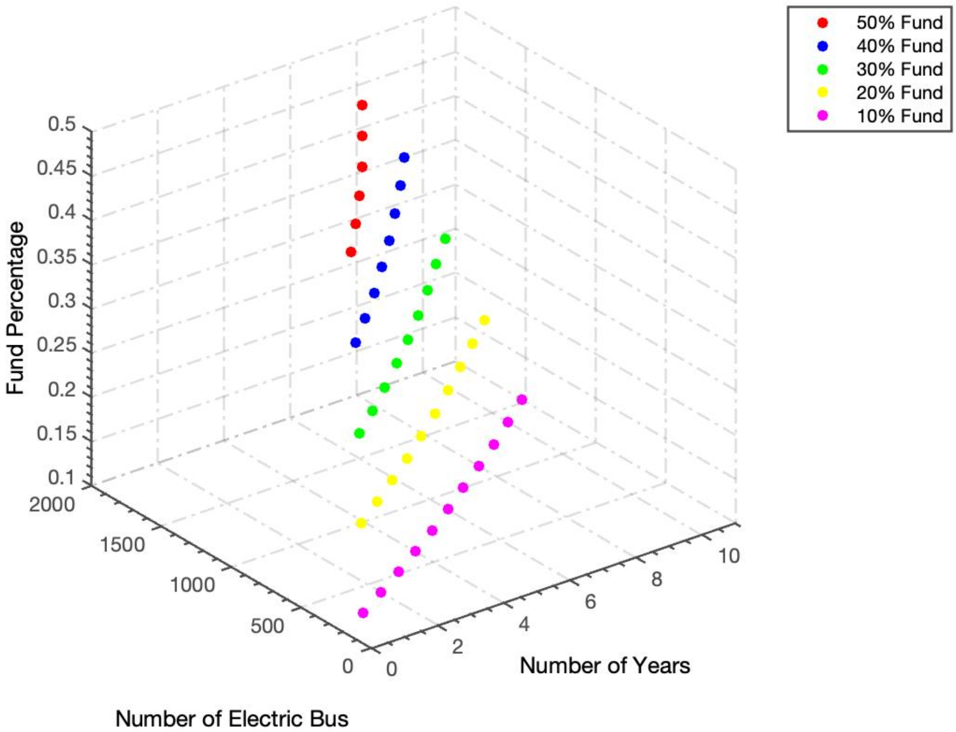

Figure 8: 3D plot of Recursive function model.

6 Problem 3: E-Bus Update Plan Model

6.1 Overall Idea

To determine the plan of electric bus transition of the next 10 years, we focus on two major factors: which bus route should transit first and where should the charging station deploy. First, we use the Financial Model from question 2 to determine the amount of electric buses that can be transited in 10 years. Based on the number of electric buses that can be transited, we then calculate how many bus routes can be transited into all-electric bus.

We then start to separate one of the priorities - which bus line should be transit first - into 3 considerations. The first, and most important, is the ridership of the bus routes. Bus routes with larger ridership should be transited into electric buses first. Second, we consider the length of the bus route. Bus routes with short lengths should be transited into electric buses first than longer bus routes. Last but not least, the elevation of the bus route should also be considered. Bus routes with a high elevation should be transit as the low priority. With 3 of the considerations, we are going to use an evaluation model to rank which should be transit as the higher priority or lower priority.

For the another major priority: deployment of a charging station, we first determine the number of charging station's by using the ration of number of charging station to electric buses. Bus routes with dense ridership should deploy more charging ports, and bus routes with high elevation should also deploy more charging ports. One charging station can hold many charging ports. We then consider the location of charging stations base on the start points and end points of bus routes. The location of bus routes should be close to the start and end points of bus routes to make it convenient for buses to charge over night. We are not considering on route charging station because it is more expensive and complex. By using clustering model to determine where the location of the stations are.

6.2 Route Priority Determination

In terms of deciding which routes should be transit into electric bus first, we consider three major factors: ridership, route length, and elevation.

6.2.1 Ridership

Bus routes with higher ridership should be transited to electric buses first because bus routes with high ridership have a slower average speed, leading to CO2 emission since they will require more energy. This means the transition from diesel buses to electric buses in high ridership bus route can maximize the advantages of electric buses' efficiency at lower velocity.

6.2.2 Route Length

As for route length, we also consider that short bus routes are usually within the dense city center and long bus routes are usually traveling from one destination to another without many bus stop. With such consideration, short bus routes' average bus speed is slower than long bus routes' average bus speed. Since electric buses have higher efficiency at a slow speed, and frequent stop allows bus to use regenerative braking, it's important to transit short bus routes first.

6.2.3 Elevation

The third factor we consider is the elevation of the bus routes. Bus routes with low elevation should be transited into electric buses first. High-elevation bus routes should transit afterward since there is a risk such as being out of charge or over capacity. This can be caused by the high consumption of batteries when buses are going up a hill or mountain. To simplify the elevation, we use 0 1 2 to shows the elevation of different bus route. For example, elevation of 2 represent bus route that go through a hill or mountain. Elevation of 1 represents the routes with elevation within 60 meter. Zero represents elevation within 30 meter.

6.3 Entropy Weight Method

We use Entropy Weight Model to determine the importance of three factors in terms of which routes should transit first. The bigger the entropy weight is, the larger the degree of dispersion is. This means the more important the factor is.

Before standardizing the data of ridership, elevation, and route length, we first have to determine whether the indicator is positive or negative impact.

If the indicator impact is positive, then a larger entropy value will accounts for a larger weight, which is standardized by the following:

$$x_{ij} = \frac{d_{ij} - \min(d_i)}{\max(d_i) - \min(d_i)} \tag{18}$$

However, if the indicator impact is negative, a smaller entropy value will accounts for a larger weight, which is standardized by the following:

$$x_{ij} = \frac{\max(d_i) - d_{ij}}{\max(d_i) - \min(d_i)} \tag{19}$$

For both equations, d_{ij} denotes the original value of ith index and jth sample, and x_{ij} denotes the standardized value. In our case, i is the bus line, and j is one of the three factors we are considering.

In our case, elevation and route length are negative indicators since the higher the elevation and the longer the route length are, the lesser we take them into consideration. Ridership is a positive indicator since the higher the amount of ridership is, the more we take it into consideration.

Next, we calculate the weight of each subject using the following:

$$p_{ij} = \frac{x_{ij}}{\sum\limits_{i=1}^{n} x_{ij}} \tag{20}$$

The entropy value E_i of ith index is calculated by the following:

$$E_i = -\frac{\sum\limits_{j=1}^{m}(p_{ij} \cdot \ln p_{ij})}{\ln n}, (0 \leq E_i \leq 1) \tag{21}$$

Then, we calculate the weight of each indicator through its entropy value:

$$d_j = 1 - e_j \tag{22}$$

$$w_j = \frac{dj}{\sum\limits_{j=1}^{m} d_j} \tag{23}$$

The overall priority index of each sample is:

$$s_i = \sum_{j=1}^{m} w_j \times P_{ij} \tag{24}$$

6.4 TOPSIS Model

To determine the order in which each bus lines will be converted to electric buses, we calculate a TOPSIS score for each line through the TOPSIS Model.

From the standardized values calculated in Entropy Weight Method, we can calculate the ideal best distance (d_{ib}) and ideal worst distance (d_{iw}) for each bus line i.

$$d_{iw} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{wj})^2} \tag{25}$$

$$d_{ib} = \sqrt{\sum_{j=1}^{n}(t_{ij} - t_{bj})^2} \tag{26}$$

where t_{ij} is the element value, and t_{wj} and t_{bj} are the ideal worst and ideal best, respectively, for the factor j. It is also important to note that if the factor has positive impact, the ideal best for j is the maximum value in the column j, and minimum value would be ideal worst. If the impact of the factor is negative, ideal best would be minimum and ideal worst would be maximum.

Then we can calculate a TOPSIS score S that indicates the solution with shortest distance from the positive ideal solution and the largest distance from the negative ideal solution.

$$S = \frac{d_{iw}}{d_{ib} + d_{iw}} \tag{27}$$

The bus line with the higher score means it has a higher rank, thus we prioritize to transit the buses on this line.

6.5 Charging Station

We first calculate the total amount of charging stations required in a city. To find out the ratio of charging station to total electric bus, we use the data from China, city of ShenZhen, as an template. This is because ShenZhen has a well develop electric bus system. According to research, ShenZhen has 15896 electric bus and 81 charging station.[16][17] By dividing total electric bus and the charging station in ShenZheng, we get a electric bus to charging station ratio, and we are able to determain the number of charging station in a city.

$$N_{CS} = TB * \frac{81}{15896} \tag{28}$$

Then by using the model from question 2, we calculate the total amount of electric buses in a city within 10 years. By multiplying the total electric bus in a city with the ratio, we get the amount of charging stations requires in a city.

We decided that all charging station is depot and ignore on-route charging. This is because on-route charging is often more expensive and easy to be damaged under harsh condition such as cold weather. Not only that, since on-route charging is along the street routes, risk such as electrical leakage may cause greater problems.

Because we will deploy depot charging station, the location of charging station needs to be close to either the start or end of the bus routes, allowing buses to charge over night.

6.5.1 K Mean Clustering Model

To determine where the charging station should be deployed, we use K mean clustering model to cluster charging station location.[18] The amount of cluster groups is equal amount of charging station in the cites. So by using k-mean clustering model, we can get the most convenient locations for charging station. The equation is shown below:

$$I = \sum_{i=1}^{m} \sum_{k=1}^{K} W_{ik} \|x^i - \mu_k\|^2 \tag{29}$$

Here, if x^i is in cluster k, then $W_ik = 1$; if it does not belongs to cluster k, then $W_ik = 0$. By using K mean cluster, we are able to determine where the charging port will be located closest to either buses start station or end station.

6.6 Result Analysis For Route

We apply the above Entropy Model, TOPSIS Model, and K Mean Clustering Model to Seattle, New York, and San Francisco.[19][20][21][22] The entropy weight result is shown in Table 4.

City	Ridership	Route Length	Elevation
Seattle	54.227%	12.272%	33.5%
San Francisco	41.216 %	14.623 %	44.1%
New York	62.115%	5.482%	32.4%

Table 5: Entropy weight for ridership, route length, and elevation in Seattle, New York, and San Francisco

6.6.1 Seattle

By using the Entropy weight model, we calculate the entropy weight of 3 factors that help determine which route should transit into an electric bus in Seattle as the priority. As shown in the table above, ridership account for 54.227% of the weight, Length accounts for 12.272%, and Elevation account for 33.5% of the weight. Since ridership accounts for half of the entropy weight, this means ridership is very important in consideration of which routes should change first. Elevation is also important because there are hills such as Capital Hill and Beacon Hill.

Bus Line	Ridership(PerWeek)	Elevation	Distance	Bus Number	Rank
RapidRide D	7666	1	14.2	102	1
RapidRide A	7716	1	18	103	2
Route 40	6383	1	12.7	85	3
RapidRide B	3305	0	16	44	4
RapidRide E	10310	2	20.1	138	5
RapidRide C	5791	1	19	77	6
Route 631	34	0	7	1	7
Route 62	4503	1	15.4	59	9
Route 45	3824	1	11.8	51	10

Table 6: Scores for each factor and overall score of Seattle

After using the Entropy Weight model, we then use the TOPSIS model to determine the priority rank for each Bus route in Seattle. The TOPSIS score for Seattle is shown in the table above, we list out the top 10 route with the highest rank. This means, this 10 bus routes is the top 10 priority bus route that should be transit into electric bus in Seattle.

6.6.2 San Francisco

Similar with Seattle, we also uses Entropy weight model for San Francisco's bus route planning. As shown in the table above, Ridership in San Francisco is accounted for 41.216% of the Entropy weight, Route length is accoutned for 14.623% of the Entropy Weight and Elevation is accounted for 44.161% of the Entropy weight. The reason why Elevation's Entropy Weight in San Francisco is almost the same with Ridership is mainly because elevation in San Francisco have a large dispersion.

Bus Line	Ridership(PerWeek)	Elevation	Distance	Bus Number	Rank
49 Van Ness/Mission	25000	1	12	81	1
38 Geary	21500	1	10.5	70	2
5 Fulton	84000	1	1	27	3
12 Folsom	6300	0	9	20	4
27 Bryant	6200	0	9.2	20	5
8 Bayshore	22800	1	18	74	6
55 Dogpatch	1900	0	4	6	7
25 Treasure Island	2800	0	8	9	8
39 Coit	500	0	4	1	9
18 46th Avenue	3200	0	20	10	10

Table 7: Scores for each factor and overall score of San Francisco

The table above shows San Franscio's TOPSIS model top 10 ranking. 10 routes represent the top 10 proirty bus route need to be transit into electric bus.

6.6.3 New York

Bus Line	Ridership(PerDay)	Elevation	Distance	Bus Number	Rank
BX12 SBS	48.124	0	7.6	103	1
BX12	48.124	0	8.1	103	2
M15 SBS	44.797	0	8.7	96	3
BX2	36.487	0	8	78	4
BX1	36.487	0	8.8	78	5
M14D	30.588	0	3.4	65	6
M14A	30.588	1	3.9	65	7
B46 SBS	43.463	1	6	93	8
B46	43.463	1	8.4	93	9
BX36	30.474	0	6	65	10

Table 8: Scores for each factor and overall score of New York

The table shown above shows the TOPSIS result for New York cities. We use TOPSIS model to rank the importance of each bus route and determine which bus routes should be transit into electric bus as the priority. The table above listed the top 10 bus route that should be transit into electric bus as the priority. This can be cause by high ridership, low elevation, or short distance.

6.7 Result Analysis for Charging Station

6.7.1 Seattle Charging Station

By using the Electric Bus to Charging Station ratio, we calculate that Seattle requires a total of 8 depot charging station. To see where is the best place to place the charging stations. We first find out the coordinate of the start and end station of each bus route in Seattle. Then by using k mean clustering model, we cluster all of the bus terminal coordinate into 8 cluster. The 8 cluster is shown below in the table and represent the 8 most convenient charging station location for all bus in Seattle.

Seattle	Latitude	Longitude
Charging Station 1	47.583345	-122.331067
Charging Station 2	47.668246	-122.279981
Charging Station 3	47.672441	-122.326911
Charging Station 4	47.600383	-122.329658
Charging Station 5	47.564496	-122.324989
Charging Station 6	47.465409	-122.280121
Charging Station 7	47.624046	-122.314998
Charging Station 8	47.677001	-122.346885

Table 9: 8 Location for Charging Station in Seattle

The eight coordinate shown above represent the recommended charging station position in Seattle.

6.7.2 San Francisco Charging Station

Through the bus charging station to bus number ratio, we calculate that San Francisco use needs 6 charging station. Due to the limitation of length, we only present the location of 3 charging stations that will be build in first 5 years. One notable result error is the charging station 2. The Charging station 2 is outside of San Francisco because it is accounting the special bus route 25 Treasure Island. Because the Charging Station 2 is only accounting 25 Treasure Island, We plan to directly build the station on the island.

San Francisco	Latitude	Longitude
Charging Station 1	37.712367	-122.136049
Charging Station 2	47.668246	-122.279981
Charging Station 3	37.750362	-122.48414

Table 10: 3 Location for Charging Station in San Francisco

6.7.3 New York City Charging Station

New York City has the largest size and the most number of bus; thus it has the most complicated public transport system needs to be convert. To avoid pages of different color pictures and data, we present the location of charging station in New York for the first two year. Overall, there are 31 charging station in New York after 10 years. In the First two year, there will be 6 charging station built. The following locations are where the charging station is.

Some errors with the location of the Charging Station is that the locations of Charging Station 1 and Charging Station 4 are located on the sea. To fix this error, we move the location onto the Shore to make the Charging Station plausible.

New York City	Latitude	Longitude(W)
Charging Station 1	40.429995	73.583237
Charging Station 2	40.873	73.90875
Charging Station 3	40.621	73.9351
Charging Station 4	40.49395	73.552675
Charging Station 5	40.835936	73.838569
Charging Station 6	40.828	73.9285

Table 11: 6 Location for Charging Station in New York

6.8 Overall planning of three cities

The following maps of three cities represent the location of the routes that needs to be change and the locations of the charging stations.

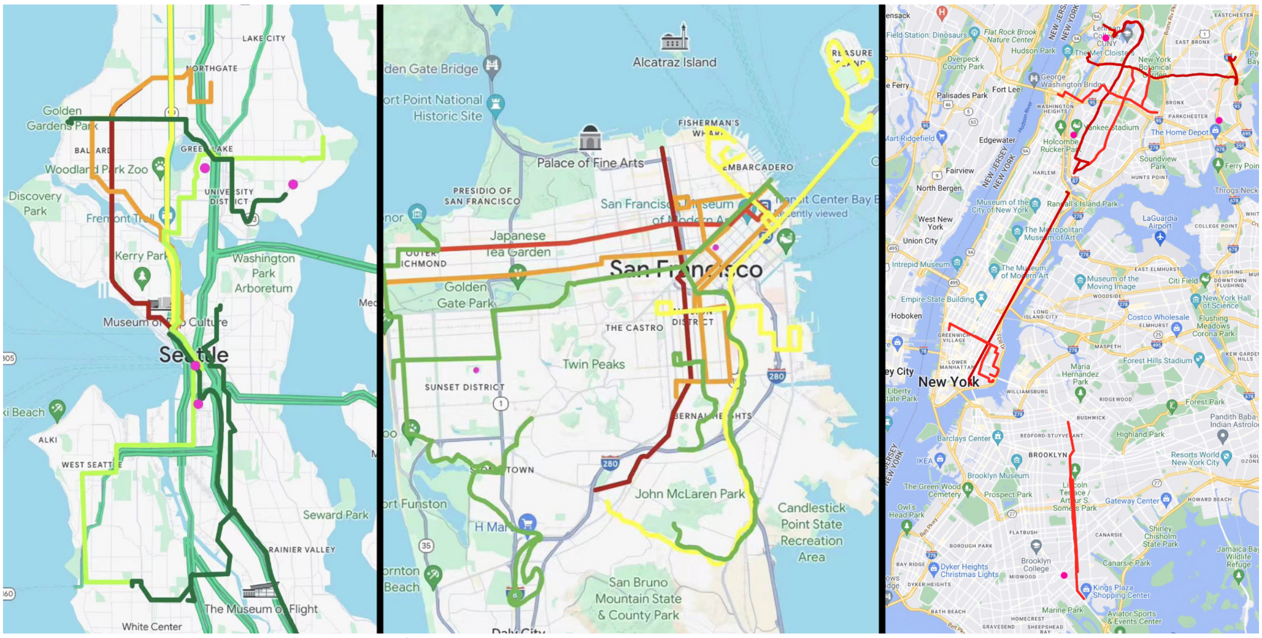

Figure 9: Seattle, San Francisco, New York E-Bus Transition Plan

The color of the routes represent the different times when the route is converted to electric bus. The dark red is the route converted in first year. Red represents second year, and orange represents the third year. Yellow represents fourth year. Finally green represents the fifth year. Pink points represent the location of charging station. For Seattle and San Francisco, we presents the first five year plan while we only present the first two year plan for New York City due to its large number of buses. A complete plan will be attach to Appendix.

7 Strengths and Weaknesses

7.1 Strengths

- For the regression model, we not only evaluate the model based on the mathematical error such as MSE, we also evaluate the model from real life limitation and developed a logistic regression that is more consistent with the real world.

- For route determination, we decided to replace the existing route with electric bus rather than make new bus stops and routes. This strategy is more practical in real life because it is cheaper for the government to operate the plan while it is convenient for the passenger.

- For route determination, we decided to replace the existing route with electric bus rather than make new bus stops and routes. This strategy is more practical in real life because it is cheaper for the government to operate the plan while it is convenient for the passenger.

7.2 Weaknesses

- Due to the limitation of length and time, we do not consider some factors that can potentially impact our model on route decision. One example is the temperature. Temperature has a potential impact on degradation of Lead ion battery. It can also affect the capacity of the electric battery. However, because temperature varies every day and weather is difficult to predict, we do not consider the factor of temperature.

- For the finance model, we are only considering one type of electric bus model, Therefore it has a constant price. However, as the electric bus technology getting more advance, there may be new models that is cheaper or more energy efficient.

8 Conclusion

In this paper, we created several models to analyze the conversion of buses to electric buses. We first analyzed the environmental consequences of converting to e-buses using regression model, and found that the number of times noise pollution is lower and the percentage of electric buses have a nonlinear, positive correlation, while the amount of CO2 emission and percentage of electric buses have a nonlinear, negative correlation. We then apply the logistic regression model for noise pollution and improved logistic regression model for CO2 emission to Seattle, and found that Seattle currently has a CO2 Emission amount of $1.416 \times 10^{1}1$, and its noise level is 1.183 compared to that of a full diesel bus fleet. When 100% of its buses are electric buses, its CO2 emission will be $6.25 \times 10^{1}0$, while its noise level will be 3.308 times lower than a full diesel bus fleet.

By examining the economic cost of converting to a full electric fleet using a recursive model, we found that it will take around 6 years for buses in Seattle to be fully converted to electric buses, if 50% of the costs are covered by external funds.

Finally, to provide 10 years plan for electric bus update, we employ the Entropy Weight Method and TOPSIS to determine which route should be transit first in Seattle, San Francisco, and New York. This is done by taking the number of ridership, route length, and elevation into consideration. We displayed the top 10 ranking of the bus route in each city. Then we used the K Mean Clustering Model to decide the location of charging stations in each city. The result shows that Seattle requires a total of 8 charging station, San Francisco requires 6, and New York requires 31 charging stations. The coordinates for the first few charging stations in each city are presented in our result analysis.

9 Letter

Date: November 14, 2023
To: Seattle Transportation Official
Subject: A Proposal for the Transition to Electric Buses in Seattle

Dear Official,

We are reaching out to initiate a plan for the transition of buses to electric buses in Seattle. As with many metropolitan areas, Seattle faces environmental challenges, particularly air pollution from transportation emissions. In our pursuit to address these concerns and mitigate the problem of climate change, we have identified a compelling solution: the adoption of electric buses.

By using the logistic regression model, we found that transitioning to electric buses can significantly contribute to the reduction of greenhouse gas emissions. According to our findings, a shift to a fully electric bus fleet allows CO2 emission to decrease from 1.416 x 10^11 g to 6.252 x 10^10 g, a change that can improve the air quality of the city and mitigate the impact of global warming. Moreover, the transition to electric buses can lower noise levels up to 3 times compared to that of a full diesel bus fleet, fostering a quieter and pleasant living environment for Seattle residents.

Given these long-term environmental and social benefits, we crafted financial models to plan the transition to electric buses. We recommend the government to cover 20% of the cost for the transition in order to implement a 10-year plan or cover up to 50% of the cost to achieve a complete electric bus fleet within 6 years.

Our recommendation involves prioritizing the conversion of bus routes based on 3 key factors: more ridership volume, shorter route length, and lower elevation. We employed the Entropy Weight Method to assign appropriate entropy weights to the indicators, and subsequently utilized the TOPSIS model to assign a priority index for each bus route in Seattle. As we aim to achieve a fully electric bus fleet by the year 2033 (10-year range), we maximize the advantage of electric buses during the transformation period.

We proposed the following plan for the transition of bus routes in Seattle for the first 5 years.

Year 1: Route D
Year 2: Route A
Year 3: Route 40 and Route B
Year 4: Route E
Year 5: Route C, Route 631, Route 62

Then, we used K mean Clustering Model to determine that Seattle requires a total of 8 depot charging stations. The following are the locations for 5 of the charging stations:

2201 3rd Avenue South, Seattle, WA 98134, USA
5251 Sand Point Way Northeast, Seattle, WA 98105, USA
124 Northeast 60th Street, Seattle, WA 98115, USA
The Lofts, 210 3rd Avenue South, Seattle, WA 98104, USA
4315 7th Avenue South, Seattle, WA 98108, USA

We hope that our plan will be helpful for the bus transition in Seattle. Thank you for your time and consideration.

Best regards

Figure 10: Letter to Transportation Officials

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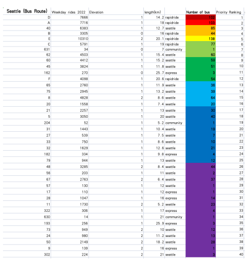
Appendices

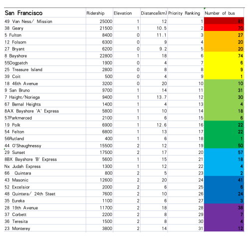
Program Code

```
public class Recuresive
{

    public static int RecModel(double f)
    {
        int bus;
        int bus_pre = 15;
        int count = 1;
        while(bus_pre < 5927)
        {
            bus = bus_pre + (int)((0.6*(311893583*2.9 -
               (5927-bus_pre)*24525.3*(0.76+1.14)
               -15*13610.7*(0.36+0.64)-(bus_pre-15)
               *24525.3*(0.36+0.64)-44501*5927))
               /((797822+0.25*60000)*(1-f)));
            System.out.println("year "+count+" bus increase
               to "+ bus);
            bus_pre = bus;
            count++;
        }
        return count;
    }
    public static void main(String[] arg)
    {
        System.out.println("50% fund");
        RecModel(0.5);
        System.out.println("40% fund");
        RecModel(0.4);System.out.println("30% fund");
        RecModel(0.3);System.out.println("20% fund");
        RecModel(0.2);System.out.println("10% fund");
        RecModel(0.1);System.out.println("0% fund");
        RecModel(0);
    }
}
```



(a) Seattle Bus Route Planning



(b) San Francisco Bus Route Planning

Figure 11: City Bus Route Planning

New York (Bus Route)	ridershi 2016 eachday	route length	elevation	Priority Ranking	Bus Number
BX12 SBS	48,124	7.6	0	1	103
BX12	48,124	8.1	0	2	103
M15	44,797	8.7	0	3	96
M15 SBS	44,797	8.7	0	3	96
BX2	36,487	8	0	4	78
BX1	36,487	8.8	0	5	78
M14D	30,558	3.4	0	6	65
M14A	30,558	3.9	0	7	65
B46 SBS	43,463	6	1	8	93
B46	43,463	8.4	1	9	93
BX36	30,474	9.4	0	10	65
BX19	29,378	6.8	0	11	63
BX15	28,156	7.6	0	12	60
B6	41,320	12.6	1	13	88
BX9	27,175	6.4	0	14	58
B44 SBS	37,418	9.4	1	15	80
B44	37,418	10.3	1	16	80
M86 SBS	24,746	2.4	0	17	53
BX40	25,454	8.2	0	18	54
BX42	25,454	8.6	0	19	54
BX6	23,800	5.8	0	20	51
BX41 SBS	23,558	5.4	0	21	50
BX41	23,558	5.5	0	22	50
M101	24,275	11.9	0	23	52
B35	31,886	7.4	1	24	68
BX35	17,651	4.1	0	25	37
Q58	29,412	10.4	1	26	63
M100	17,006	7.3	0	27	36
BX39	16,434	7.6	0	28	35
M60 SBS	16,751	9.4	0	29	35
M96	15,034	2	0	30	32
M34 SBS	14,901	2.1	0	31	31
M34A SBS	14,901	2.5	0	32	31
B82	27,959	12.8	1	33	59
BX38	15,750	8.8	0	34	33
BX3	14,921	4.5	0	35	32
BX21	15,234	6.9	0	36	32
BX28	15,750	9.7	0	37	33
M4	16,235	12.1	0	38	34
B41	27,082	10.5	1	39	58
M79 SBS	13,892	2.2	0	40	29
BX22	14,690	6.6	0	41	31
Q44 SBS	27,712	14.4	1	42	59
BX11	14,269	4.9	0	43	30
BX7	13,720	6	0	44	29
M23 SBS	12,308	2.6	0	45	26
M7	12,905	7.8	0	46	27
BX13	12,094	4.1	0	47	25
M3	13,149	10.5	0	48	28
M66	11,195	2	0	49	24
M102	12,292	7.7	0	50	26
BX4	11,411	4.7	0	51	24
BX4A	11,411	4.9	0	52	24
M42	10,596	2.1	0	53	22
M116	10,296	2.6	0	54	22
BX10	11,266	7.4	0	55	24
Q27	24,580	11.9	1	56	52
BX5	11,194	7.7	0	57	24
M31	9,995	3.9	0	58	21
M55	10,063	4.5	0	59	21
BX17	10,219	5.7	0	60	21

(a) New York Bus Route Planning 1

Q27	24,580	11.9	1	56	52
BX5	11,194	7.7	0	57	24
M31	9,995	3.9	0	58	21
M55	10,063	4.5	0	59	21
BX17	10,219	5.7	0	60	21
BX31	10,019	6.4	0	61	21
M11	10,152	7.9	0	62	21
M1	10,436	9.4	0	63	22
BX30	9,680	7.3	0	64	20
M103	9,786	7.8	0	65	20
M5	10,063	9.4	0	66	21
Q10*	23,363	11.2	1	67	50
M104	8,223	5.2	0	68	17
M2	9,275	10.5	0	69	19
BX27	7,435	3.6	0	70	15
BX26	7,988	6.1	0	71	17
BX16	8,239	7.4	0	72	17
M10	6,874	5.2	0	73	14
BX32	6,809	5.2	0	74	14
M57	6,065	3	0	75	13
B15	22,282	13.3	1	76	47
BX8	7,282	8.7	0	77	15
M72	5,675	3	0	78	12
Q46	21,468	10.2	1	79	46
M9	5,333	4.8	0	80	11
Q65*	21,052	10.4	1	81	45
BX34	4,268	3.9	0	82	9
B1	19,828	6.8	1	83	42
Q25*	20,083	9.5	1	84	43
M50	2,785	2.3	0	85	5
BX33	2,866	2.9	0	86	6
M22	2,732	2.8	0	87	5
BX23*	3,829	6.5	0	88	8
M106	2,476	2.5	0	89	5
M8	2,107	2.2	0	90	4
BX18	1,920	2.4	0	91	4
B8	19,616	10	1	92	42
B38	19,099	8	1	93	40
M21	1,402	2.4	0	94	3
BX29	2,476	5.9	0	95	5
M20	2,446	6.4	0	96	5
M35	1,038	3	0	97	2
BX46	619	2.6	0	98	1
BX20	772	3.7	0	99	1
M98	1,863	7.3	0	100	3
M12	515	3.5	0	101	1
BX24	754	4.5	0	102	1
Q17	17,224	8.7	1	103	36
B12	15,380	4.3	1	104	33
Q23*	16,059	6.9	1	105	34
Q43	14,853	7.8	1	106	31
B36	14,026	5.8	1	107	30
B9	14,404	8	1	108	30
B68	14,159	7.6	1	109	30

(b) New York Bus Route Planning 2

Figure 12: New York Bus Route Planning

B68	14,159	7.6	1	109	30
Q66*	13,824	7.4	1	110	29
Q20A	14,192	9	1	111	30
Q20B	14,192	9.2	1	112	30
B52	12,435	5.1	1	113	26
B103*	14,448	11.8	1	114	31
Q53*	14,752	14.4	1	115	31
Q60*	13,761	10.9	1	116	29
B3	11,803	5.3	1	117	25
B11	11,273	5.8	1	118	24
Q06*	11,334	6.2	1	119	24
Q12	11,099	5.8	1	120	23
Q64*	9,731	2.6	1	121	20
B54	10,466	4.4	1	122	22
B17	11,279	6.5	1	123	24
Q111*	12,390	10.3	1	124	26
B49	12,452	10.7	1	125	26
B63	11,607	8.2	1	126	24
Q5	11,728	8.6	1	127	25
Q49*	9,303	2.6	1	128	19
Q54	11,952	9.7	1	129	25
Q85	11,820	9.6	1	130	25
Q69*	9,849	5.1	1	131	21
B43	10,540	6.8	1	132	22
Q28	9,893	5.3	1	133	21
B25	10,048	6	1	134	21
B61	9,945	5.8	1	135	21
B26	9,723	5.9	1	136	20
Q08*	10,696	8.4	1	137	22
B47	10,436	7.8	1	138	22
Q30	10,555	8.5	1	139	22
Q88	11,010	9.8	1	140	23
Q33*	7,773	2.5	1	141	16
B60	9,906	7.9	1	142	21
Q4	9,242	6.6	1	143	19
Q18*	8,780	5.6	1	144	18
Q32	9,454	7.4	1	145	20
Q3	9,684	8.5	1	146	20
Q13	8,516	6.3	1	147	18
B83	8,281	6	1	148	17
Q83	8,664	7	1	149	18
Q55	7,343	4.4	1	150	15
Q37*	7,842	5.9	1	151	16
B14	7,243	5.2	1	152	15
Q56	7,862	6.6	1	153	16
Q47*	8,043	7	1	154	17
Q38*	8,465	8	1	155	18
Q24	8,512	8.5	1	156	18
Q72*	6,204	4.3	1	157	13
B62	7,882	7.6	1	158	16
B42	4,186	1.3	1	159	8
B45	6,019	4.6	1	160	12
Q15	5,876	4.4	1	161	12
Q40*	4,976	3	1	162	10

(a) New York Bus Route Planning 3

Q40*	4,976	3	1	162	10
Q110*	6,281	5.3	1	163	13
Q09*	5,050	3.3	1	164	10
B70	5,942	4.8	1	165	12
Q15A	5,876	4.7	1	166	12
Q2	5,722	4.6	1	167	12
Q29*	5,402	4.1	1	168	11
Q59	7,127	7.3	1	169	15
Q112*	5,217	4	1	170	11
Q34*	6,941	7.1	1	171	14
B100*	5,387	4.4	1	172	11
B20	7,042	7.5	1	173	15
B57	7,244	8	1	174	15
B74	4,225	3	1	175	9
Q77	6,413	6.7	1	176	13
Q41*	7,204	8.9	1	177	15
Q22*	7,247	9	1	178	15
B16	6,739	8	1	179	14
B7	6,066	6.8	1	180	13
B64	5,732	6.9	1	181	12
B31	2,753	2.9	1	182	5
B2	2,464	2.6	1	183	5
Q39*	5,548	7.1	1	184	11
Q84	4,441	5.4	1	185	9
Q07*	5,341	6.9	1	186	11
B13	6,290	8.8	1	187	13
Q104*	2,296	2.8	1	188	4
B65	3,398	4.5	1	189	7
Q1	4,381	5.9	1	190	9
B4	6,195	9.2	1	191	13
Q100*	4,129	5.7	1	192	8
Q31	5,692	8.6	1	193	12
B69	4,225	6.2	1	194	9
Q42	1,195	2.5	1	195	2
Q101*	3,632	5.4	1	196	7
B39	260	1.8	1	197	1
Q16	4,054	6.2	1	198	8
B84	595	2.2	1	199	1
B48	3,851	6	1	200	8
B67	4,504	7.1	1	201	9
Q36	6,101	10.2	1	202	13
Q26	1,466	3.5	1	203	3
Q103*	1,385	3.5	1	204	2
Q11*	5,004	8.4	1	205	10
Q19*	3,467	6.3	1	206	7
Q76	5,742	10.1	1	207	12
Q48	2,854	5.6	1	208	6
Q102*	2,663	5.6	1	209	5
B32	819	3.6	1	210	1
Q35*	4,426	8.9	1	211	9
Q114*	6,396	13.2	1	212	13
Q70 SBS*	4,183	8.8	1	213	8
B37	2,380	6.3	1	214	5
B24	2,390	6.5	1	215	5

(b) New York Bus Route Planning 4

B24	2,390	6.5	1	215	5
Q67*	2,768	7	1	216	5
Q113*	4,569	10.3	1	217	9
Q21*	2,926	8.4	1	218	6
Q52*	5,356	13.1	1	219	11
Q50*	4,690	12.3	1	220	10
S48	7,702	6.8	2	221	16
S53	9,378	9.4	2	222	20
S79 SBS	11,378	15.1	2	223	24
S42	914	3.4	2	224	1
S96	7,299	9.6	2	225	15
S46	7,299	10	2	226	15
S94	7,016	9.9	2	227	15
S40	4,629	7.8	2	228	9
S90	4,629	7.8	2	228	9
S44	7,016	10.9	2	229	15
S62	4,431	8.4	2	230	9
S92	4,431	8.4	2	230	9
S51	4,169	8.7	2	231	8
S93	3,686	8.8	2	232	7
S76	4,617	9.6	2	233	9
S91	4,385	9.5	2	234	9
S52	3,450	9	2	235	7
S61	4,385	9.8	2	236	9
S66	1,659	8.4	2	237	3
S56	515	9.2	2	238	1
S55	355	10.5	2	239	0
S57	1,436	11.3	2	240	3
S54	1,315	11.5	2	241	2
S89	756	11.8	2	242	1
S78	6,163	20.8	2	243	13
S74	5,411	19.4	2	244	11
S59	3,592	16	2	245	7

(c) New York Bus Route Planning 5

Figure 13: New York Bus Route Planning